

Combinatorial Circuits

Mechanical and Electrical Engineering

Second Grade Level

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Combinatorial Circuits (1)

- Design of Combinatorial Circuits
 - Problem: Please add two bits
 - Hints
 - If you add two numbers a carry can occur
 - You need not only determine the result but also the carry
 - On the other hand there can already be a carry when you add two numbers
 - You have to handle this carry, too
 - This type of circuit is called full adder
 - Mathematical description of a full adder
 - $\{0,1\} \times \{0,1\} \times \{0,1\} \mapsto \{0,1\} \times \{0,1\}$
 $(a,b,c_i) \rightarrow (c_o,r)$
 c_i : carry in; c_o : carry out; r : LSB of $a+b$ (*left most bit*)

Or for short:
 $\{0,1\}^3 \mapsto \{0,1\}^2$

Combinatorial Circuits (2)

- Design of Combinatorial Circuits (continued)
 - Example of a Binary Addition

Bit position	3	2	1	0
1st Number (7)	0	1	1	1
2nd Number (5)	0	1	0	1
Carry	1	1	1	0
Result (12)	1	1	0	0

Combinatorial Circuits (3)

- Design of Combinatorial Circuits (continued)
 - Truth table of a one bit full adder

a	b	c_i	c_o	y
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Combinatorial Circuits (4)

- Design of Combinatorial Circuits (continued)
 - Switching function $y(a,b,c_i)$

		a	
	1		1
1		1	
		c _i	
			b

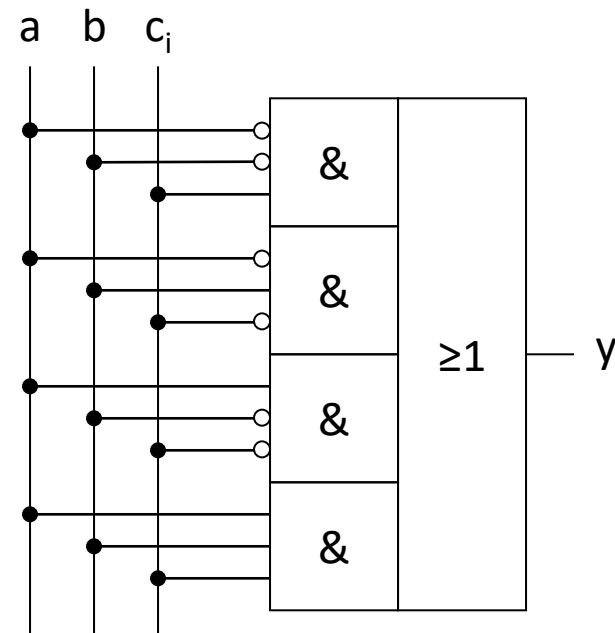
$$t_0 = \neg a \wedge \neg b \wedge c_i$$

$$t_1 = \neg a \wedge b \wedge \neg c_i$$

$$t_2 = a \wedge \neg b \wedge \neg c_i$$

$$t_3 = a \wedge b \wedge c_i$$

$$y = t_0 \vee t_1 \vee t_2 \vee t_3$$



Combinatorial Circuits (5)

- Design of Combinatorial Circuits (continued)
 - Switching function $c_o(a,b,c_i)$

		a	
		1	
	1	1	1

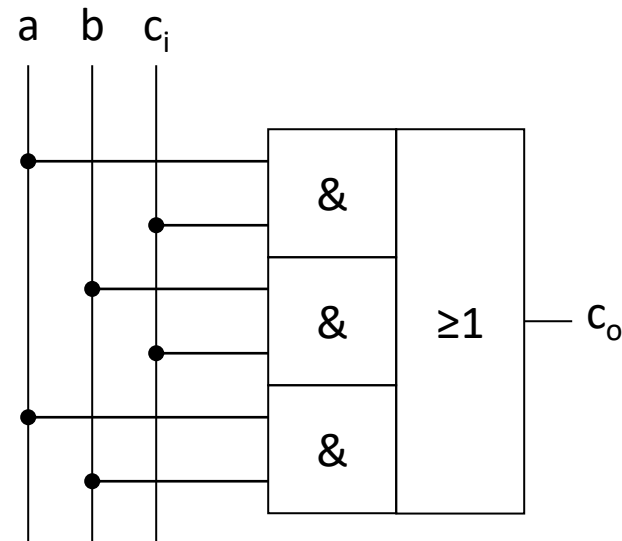
c_i b

$$t_0 = a \wedge c_i$$

$$t_1 = b \wedge c_i$$

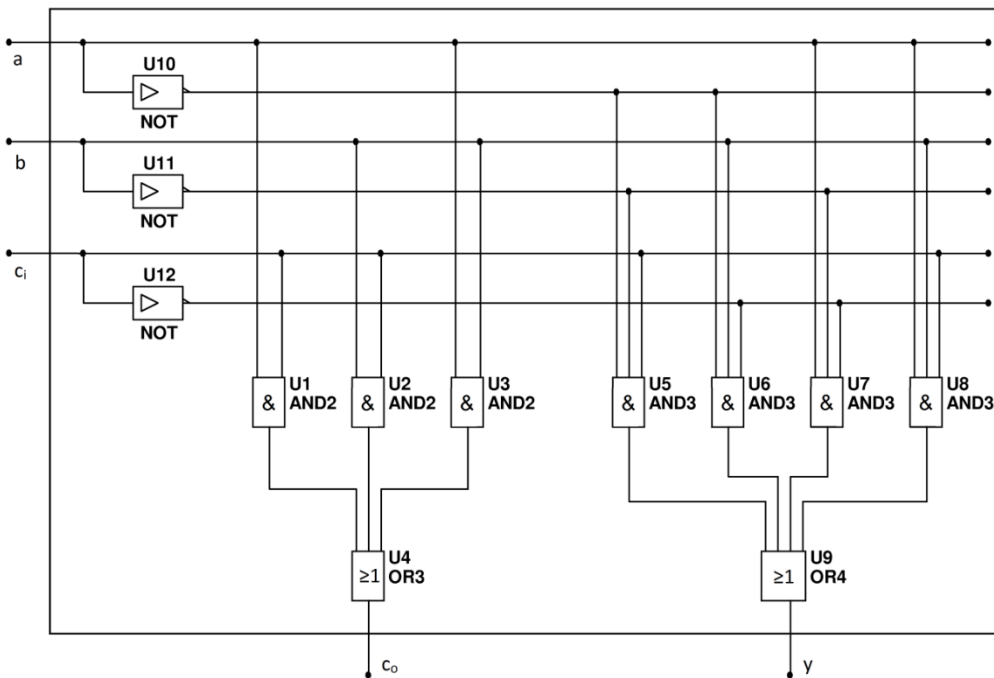
$$t_2 = a \wedge b$$

$$c_o = t_0 \vee t_1 \vee t_2$$

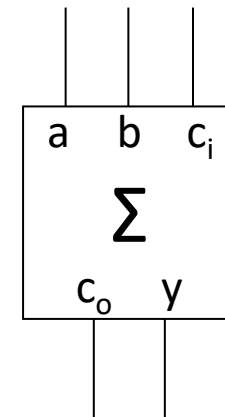


Combinatorial Circuits (6)

- Design of Combinatorial Circuits (continued)
 - Both functions integrated in a circuit

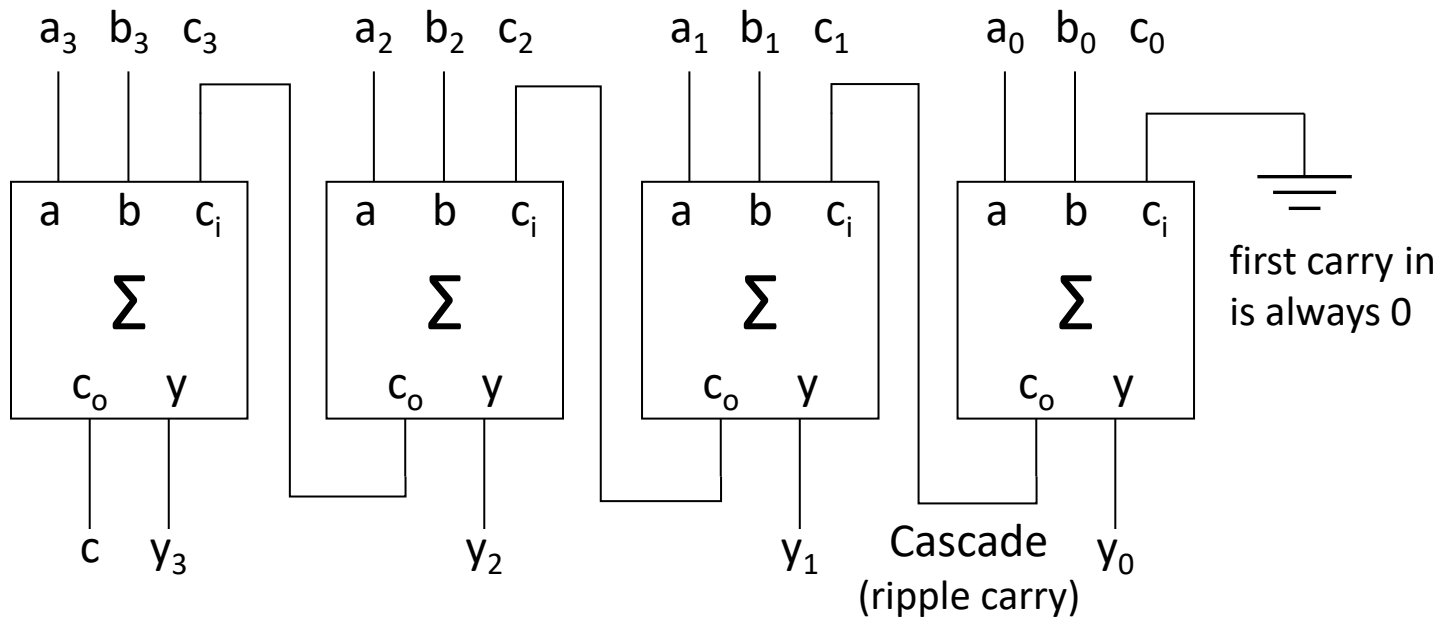


One-Bit Full Adder



Combinatorial Circuits (7)

- Design of Combinatorial Circuits (continued)
 - Four-bit Full Adder
 - Cascade of four one-bit full adders



Combinatorial Circuits (8)

- Design of Combinatorial Circuits (finished)
 - Integrated four bit half adder

